

正多面体 (星型合) の体積比

小星型 12 面体の体積 (辺長 1 の正 20 面体に内接)

$$\begin{aligned} \frac{5\varphi^{-2}}{2} &= \frac{5(2-\varphi)}{2} \approx \frac{5(2-1.618)}{2} \approx \frac{5 \times 0.382}{2} \approx 0.955 \\ &\Downarrow \\ &= \frac{5(2-\varphi)}{2} = \frac{10-5\varphi}{2} = 5 - \frac{5}{2} \times \frac{1+\sqrt{5}}{2} = \frac{20-5-5\sqrt{5}}{4} = \frac{15-5\sqrt{5}}{4} \end{aligned}$$



大 12 面体の体積 (辺長 1 の正 20 面体に内接)

$$\begin{aligned} \frac{5\varphi^{-1}}{2} &= \frac{5(\varphi-1)}{2} \approx \frac{5 \times 0.618}{2} \approx 1.545 \\ &\Downarrow \\ &= \frac{5(\varphi-1)}{2} = \frac{5\varphi-5}{2} = \frac{5}{2} \times \frac{1+\sqrt{5}}{2} = \frac{5}{2} \times \frac{5+5\sqrt{5}}{2} = \frac{10}{4} = \frac{5\sqrt{5}-5}{4} \end{aligned}$$



大星型 12 面体 (辺長 1 の正 12 面体に内接)

$$\begin{aligned} \frac{5\varphi^{-1}}{2} &= \frac{5(\varphi-1)}{2} \approx \frac{5 \times 0.618}{2} \approx 1.545 \\ &\Downarrow \\ &= \frac{5(\varphi-1)}{2} = \frac{5\varphi-5}{2} = \frac{5}{2} \times \frac{1+\sqrt{5}}{2} = \frac{5}{2} \times \frac{5+5\sqrt{5}}{2} = \frac{10}{4} = \frac{5\sqrt{5}-5}{4} \end{aligned}$$



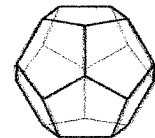
大 20 面体の体積 (辺長 1 の正 20 面体に内接)

$$\begin{aligned} \frac{32-19\varphi}{2} &= \frac{13-19\varphi^{-1}}{2} = \frac{13-(19-19\varphi^2)}{2} = \frac{19\varphi^2-6}{2} = \frac{19\varphi^2-6(\varphi^{-1}+\varphi^2)}{2} = \frac{13\varphi^2-6\varphi^{-1}}{2} = \frac{\varphi^2(13-6\varphi)}{2} = \frac{\varphi^2(7-6\varphi^{-1})}{2} = \frac{\varphi^2\left\{7-6\left(\frac{\sqrt{5}-1}{2}\right)\right\}}{2} \\ &= \frac{\varphi^2(10-3\sqrt{5})}{2} = \frac{\varphi^2(10-3\sqrt{5})(10+3\sqrt{5})}{2(10+3\sqrt{5})} = \frac{\varphi^2(100-45)}{2(7+3(1+\sqrt{5}))} = \frac{55\varphi^2}{2(7+3 \times 2\varphi)} = \frac{55\varphi^2}{2(7+6\varphi)} = \frac{55\varphi^2}{2(k^2+k)} \\ &\Downarrow \\ \frac{32-20\varphi+\varphi}{2} &= 16-10\varphi+\frac{\varphi}{2} = 16-16.18+\frac{1.618}{2} = 0.809-0.18 = 0.629 \end{aligned}$$



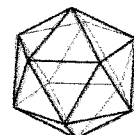
正 12 面体の体積 (辺長 1)

$$\frac{\varphi^3 \cdot k}{2} = \frac{\varphi^3(\varphi+2)}{2} = \frac{\varphi^4+2\varphi^3}{2} = \frac{\varphi^4+\varphi^3+\varphi^3}{2} = \frac{\varphi^4+\varphi^3+\varphi^2+\varphi}{2}$$



正 20 面体の体積 (辺長 1)

$$\begin{aligned} \frac{k^2}{6} &= \frac{5\varphi^2}{6} = \frac{3\varphi^2+2\varphi^2}{6} = \frac{(\varphi^2+\varphi^2)\varphi^2+2\varphi^2}{6} = \frac{\varphi^4+1+2\varphi^2}{6} = \frac{\varphi^4+1+\varphi^2+\varphi^2+(\varphi+\varphi^{-1})-(\varphi+\varphi^{-1})}{6} = \frac{\varphi^4+\varphi^2+\varphi+\varphi^2+\varphi^{-1}+1-(\varphi+\varphi^{-1})}{6} \\ &= \frac{\varphi^4+\varphi^3+\varphi^2+\varphi-\sqrt{5}}{6} \end{aligned}$$

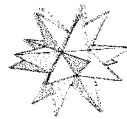
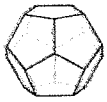


$$\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339 \quad \sqrt{5} = \varphi + \varphi^{-1} = 1 + 2\varphi^{-1} \quad k = \varphi + 2 = \varphi^2 + 1 = \varphi\sqrt{5} = 3.6180339$$

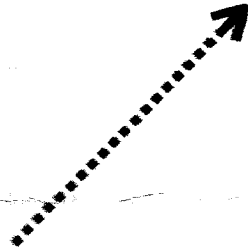
体積比較

(正多面体は辺長1、星型正多面体は前記の正多面体に内接する)

正12面体 : 正20面体 : 小星型12面体 : 大12面体 : 大星型12面体 : 大20面体



	$\frac{\varphi^3 k}{2}$:	$\frac{k^2}{6}$:	$\frac{5\varphi^{-2}}{2}$:	$\frac{5\varphi^{-1}}{2}$:	$\frac{5\varphi^{-1}}{2}$:	$\frac{55\varphi^{-2}}{2(k^2+k)}$
	\Downarrow		\Downarrow		\Downarrow		\Downarrow		\Downarrow		\Downarrow
$\frac{2\varphi^2}{5} \times$	$\frac{\varphi^2 \cdot \varphi^3 k}{5}$		$\frac{\varphi^2 k^2}{15}$		1		φ		φ		$\frac{11}{k^2+k}$
	\Downarrow		\Downarrow		\Downarrow		\Downarrow		\Downarrow		\Downarrow
	$\frac{\varphi^2(\varphi^4 + \varphi^3 + \varphi^2 + \varphi)}{5}$		$\frac{\varphi^2(\varphi^4 + \varphi^3 + \varphi^2 + \varphi - \sqrt{5})}{15}$		1		φ		φ		$\frac{11}{\varphi^4 + \varphi^3 + \varphi^2 + \varphi + \varphi^{-1} + \varphi^{-2} + \varphi^{-3} + \varphi^{-4}}$



$$\begin{aligned} \downarrow 7 &= \varphi^4 + \varphi^{-4} \\ k^2 + k &= 5\varphi^2 + \varphi + 2 = 7 + 6\varphi = \varphi^4 + \varphi^{-4} + 6\varphi = \varphi^4 + 3\varphi + 3\varphi + \varphi^{-4} = \varphi^4 + 3\varphi + \varphi(\varphi^2 + \varphi^{-2}) + \varphi^{-4} = \varphi^4 + 3\varphi + \varphi^3 + \varphi^{-1} + \varphi^{-4} \\ &= \varphi^4 + \varphi^3 + 3\varphi + \varphi^{-2} + \varphi^{-3} + \varphi^{-4} = \varphi^4 + \varphi^3 + \varphi + \varphi + \varphi + \varphi^{-2} + \varphi^{-3} + \varphi^{-4} = \varphi^4 + \varphi^3 + \varphi + \varphi + 1 + \varphi^{-1} + \varphi^{-2} + \varphi^{-3} + \varphi^{-4} \\ &= \varphi^4 + \varphi^3 + \varphi^2 + \varphi + \varphi^{-1} + \varphi^{-2} + \varphi^{-3} + \varphi^{-4} \end{aligned}$$

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(全て小文字、苗字と名前の間に半角以上のスペースを入れる)

osamu koyana